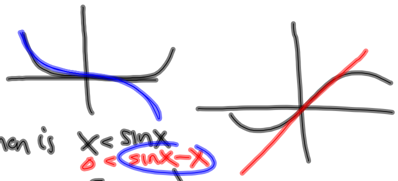
21. $\sin x \approx x \qquad |x| < 10^{-3}$

$\epsilon = |\sin x - x|$ max at $.001$ $= 1.67\times10^{-10}$



when is $x < \sin x$

$0 < \sin x - x$

$[-.001, 0)$

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33. $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \qquad |x| < 0.1$

$-0.1 \le x \le 0.1$

Approx error

Taylor's remainder thm

$$\epsilon = \frac{M(x-a)^{n+1}}{(n+1)!} = \frac{M x^4}{4!} \qquad M = \text{Max of } f^{4} = \text{Max of } e^x$$

$$\epsilon = \frac{e^{0.1}x^4}{4!} \text{ max at } x = 0.1 \qquad M = e^{0.1}$$

$$= \frac{e^{0.1}(0.1)^4}{4!} =$$

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9.4a Tests for Convergence of Series

nth term test for divergence

If seq. does not converge to 0, then the series diverges

geo converge if $|r| < 1$

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots = \frac{1}{1-\frac{1}{2}} = 2$$

series converges to 2

seq. converges to 0

Does the series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{n+1}{n} = \frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} \cdots$$

series diverges

seq. converges to 1

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Harmonic Series (diverges)

$$1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}} \cdots$$

compare

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$$

diverges

seq. converges to 0

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The direct comparison test.

If $0 < a_n < b_n$ and $\sum b_n$ converges then $\sum a_n$ also converges
sequences series

If $a_n > b_n > 0$ and $\sum b_n$ diverges then $\sum a_n$ also diverges

Does the series converge or diverge?

$$\sum_{n=0}^{\infty} \frac{3^n}{5^n+1} < \sum_{n=0}^{\infty} \frac{3^n}{5^n} = 1 + \frac{3}{5} + \frac{9}{25} + \frac{27}{125} \dots$$

its geo $r = \frac{3}{5}$

converges

converges by comparision

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Absolute convergence

If $\sum |a_n|$ converges then $\sum a_n$ converges

Show the series converges for all x

$$\sum_{n=0}^{\infty} \left| \frac{(\sin(x))^n}{n!} \right| \le \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \dots$$

converges to e

converges by comparison

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$$

let $x = 1$

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The Ratio Test works on series with factorials or exponents

given $\sum a_n$, let $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

If $L < 1$, series converges
If $L > 1$, series diverges
If $L = 1$, test fails

Does the series converge or diverge?

$$\sum_{n=0}^{\infty} \frac{3^n}{n!}$$

converges

$$\lim_{n\to\infty} \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}}$$

???

$$\frac{3^{n+1}}{3^n} = 3$$

$$\lim_{n\to\infty} \frac{\cancel{3^{n+1}}}{(n+1)!} \cdot \frac{n!}{\cancel{3^n}} = \lim_{n\to\infty} \frac{3 \cdot \cancel{n} \cdot \cancel{(n-1)} \cdot \cancel{(n-2)} \dots \cancel{1}}{(n+1) \cdot \cancel{n} \cancel{(n-1)} \cancel{(n-2)} \dots \cancel{1}}$$

$$= \lim_{n\to\infty} \frac{3}{n+1} = 0$$

$L = 0$

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do 9.4: 29-45 all

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